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NSM Solutions of Free convective flow from a Non-Isothermal Vertical Cone

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Abstract: Natural convection effects of the numerical solution for unsteady, laminar, free convection flow over an incompressible viscous fluid past a non-isothermal vertical cone with surface temperature varying as power function of distance from the apex is presented here. The dimensionless governing equations of the flow that are unsteady, coupled and non-linear partial differential equations are solved using the Network Simulation Model (NSM), a robust numerical technique which demonstrates high efficiency and accuracy and the computer code Pspice. The velocity and temperature fields have been studied for various parameters Prandtl number, semi vertical angle and n. The present results are compared with available results in literature and are found to be in good agreement.
Keywords: Free convection, Non-uniform surface temperature, NSM, PSPICE, Unsteady, Vertical cone.

Introduction

Natural convection flows under influence of gravitational force have been investigated most extensively because they occur frequently in nature as well as in science and engineering applications. When a heated surface is in contact with the fluid, the result of temperature difference causes buoyancy force, which induces the natural convection. The atmospheric circulation with all its hurricanes, blizzards and monsoons are essentially driven by natural convection. Mainly, these types of heat transfer problems deal with the design of spacecrafts, nuclear reactor, solar power collectors, power transformers, steam generators etc.

Since 1953, several authors have developed similarity solutions for axi-symmetrical problems for natural convection laminar flow over a vertical cone in steady state. [1,2]developed the general relation for similar solutions on iso-thermal axi-symmetric forms and they showed that the vertical cone has such a solution in steady state. [3]obtained similar solutions for isothermal axi-symmetric bodies (i.e., cone, parabolic-nosed, flat-nosed bodies) with closed lower ends, and integral methods are used for obtaining heat transfer results for a wide range of Prandtl numbers. These authors also presented the results obtained by numerically integrating the differential equations with including Prandtl number of 0.72 and concluded; the body shape influences the heat transfer strongly for lower Prandtl numbers. Further, [4]showed the similarity solutions exist for steady free convection flow over a vertical cone with variable surface temperature and it varies as power function of distance from apex along the cone ray. Numerical solutions of the transformed boundary layer equations are obtained for both isothermal and linear surface temperature with Prandtl number 0.7. They noticed from the velocity and temperature profiles that the dimensionless tangential-flow function for the iso-thermal cone attains 22% greater than that for the cone with linear surface temperature distribution. [5]extended the problem of [4] for low Prandtl number fluids, and obtained numerical solutions for liquid metals and concluded that the thermal boundary layer thickness is more for low Prandtl number fluids.[6]observed that the boundary layer thickness in the case of iso-thermal vertical plate for air ($\Pr = 0.733$) is comparatively 14.5 times less than that for liquid sodium ($\Pr = 0.003$).[7] Extended the work of[4] for high Prandtl number fluids, and derived expressions for local skin-friction and Nusselt numbers. Also, [8]has investigated the overall heat transfer in laminar natural convection from vertical cones using the integral method. [9] Have studied the compressibility effects in laminar free convection from a vertical cone. [10] Who investigated the steady mixed convection flow over a vertical cone for two values of the Pr, namely Pr=0.733 (air) and Pr=6.7 (water) However, these authors have considered only the case of assisting flow. Recently, [11] analyzed the steady laminar mixed convection boundary-layer flow over a vertical isothermal cone for fluids of any $\Pr$ for the both cases of buoyancy assisting and buoyancy opposing flow conditions. The resulting non-similarity boundary-layer equations are solved numerically using the Keller-box scheme for fluids of any $\Pr$ from very small to extremely large values $(0.001 \le \Pr \le 10000)$.

Recently, [12] studied the non-similarity solutions for the laminar free convection from a vertical permeable cone with non-uniform surface temperature. Using a finite difference method, a series solution method and asymptotic solution method, the solutions have been obtained for the non-similarity boundary layer equations. [13] investigated the effect of thermo physical quantities on the free convection flow of gases over iso-thermal vertical cone in steady state, in which thermal conductivity, dynamic viscosity and specific heat at constant pressure were to be assumed a power law variation with absolute temperature. They concluded the heat transfer increases with suction and decreases with injection.

To solve the present problem a well-tested, highly adaptive numerical procedure known as the Network Simulation Method (NSM) has been applied. This method was developed originally for semiconductor applications by[14] at the University of California at Berkeley.It has subsequently been implemented successfully in a wide spectrum of engineering problems. NSM is based on the classical thermoelectric analogy between thermal and electrical variables. NSM can be used for any kind of non-linearity due to boundary conditions, temperature differences of the thermal properties etc. This technique has been employed quite recently to complex nonlinear thermo fluid dynamic problems. Different reports reveal formulated in its present form by utilizing a discretization procedure for the differential equations. NSM takes the partial differential equations that define the mathematical model of the physical process and by means of spatial discretization [15], yields the ordinary differential equations which are the basis for implementing the standard electrical network model for an elemental control volume. Time remains as a continuous variable in the discretized equations. Based on these equations, a network model is designed. A number of networks are connected in series to make up the whole medium and boundary conditions are added by means of special electrical devices. The main advantage (even for complicated non-linear problems) is that the network model is comprises by very few electrical devices connected in series to which the boundary conditions are added to form the whole model of the medium. Network model designed here is the combinations of resistors, current control generators and capacitors with minimum programming rules. Once the complete network model is designed, a computer code Pspice[14] is used to simulate it and to provide the numerical solution. [16] Represents the relation between NSM and heat transfer. Many researchers [17,18,19,20,21,22,23] have solved different problems of free convection using the Network simulation method.

The investigation, namely unsteady laminar natural convection flow past an non- isothermal vertical cone solved by using network simulation method has not received any attention in literature. Hence, the present work-studies and deals with the laminar free convection flow over a non- isothermal vertical cone.

Mathematical Analysis

An axi-symmetric unsteady laminar free convection of a viscous incompressible flow past a vertical cone with variable temperature $T_w^{'}(x) = T_\infty^{'} + a\,x^n$ on the surface is considered. It is assumed that the viscous dissipation effects are negligible. It is also assumed that the cone surface and the surrounding fluid which is at rest are at the same temperature $T_\infty^{'}$. Then at time $t' > 0$, the temperature of the cone surface is suddenly raised to $T_w^{'}(x) = T_\infty^{'} + a\,x^n$ and it is maintained.

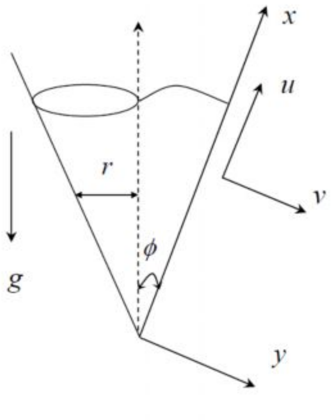

Fig. 1. Physical model and co-ordinate system

The co-ordinate system is chosen (as shown in Fig.1) such that x measures the distance along surface of the cone from the apex $(x=0)$, and y measures the distance normally outward. Here, ϕ is the semi vertical angle of the cone and r is the local radius of the cone. The fluid properties are assumed constant except for density variations, which induce buoyancy force term in the momentum equation. The governing boundary layer equations of continuity, momentum and energy under Boussinesq approximation are as follows:

$$\frac{\partial(ur)}{\partial x}+\frac{\partial(vr)}{\partial y}=0, \qquad (1)$$

$$\frac{\partial u}{\partial t'}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}= g\beta\cos\phi\left(T'-T'_{\infty}\right)+\upsilon\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2}. \qquad (3)$$

The initial and boundary conditions are

$$\begin{aligned} &t'\le 0: u=0,\ v=0,\ T'=T'_{\infty}\ \text{for all } x \text{ and } y,\\ &t'>0: u=0,\ v=0,\ T'(x)=T'_{\infty}+a x^{n}\ \text{at } y=0,\\ &\quad u=0,\quad T'=T'_{\infty}\quad \text{at } x=0,\\ &\quad u\to 0,\quad T'\to T'_{\infty}\quad \text{as } y\to\infty \end{aligned} \qquad (4)$$

Using the following non-dimensional quantities:

$$\begin{aligned} &X=\frac{x}{L},\ Y=\frac{y}{L}(Gr_L)^{\frac{1}{4}},\ R=\frac{r}{L},\ \text{where } r=x\sin\phi\\ &V=\frac{vL}{\upsilon}(Gr_L)^{\frac{-1}{4}},\ U=\frac{uL}{\upsilon}(Gr_L)^{\frac{-1}{2}},\ t=\frac{\upsilon t'}{L^2}(Gr_L)^{\frac{1}{2}},\\ &T=\frac{(T'-T'_{\infty})}{T'_w(L)-T'_{\infty}},\ Gr_L=\frac{g\beta\left(T'_w(L)-T'_{\infty}\right)L^3}{\upsilon^2},\ \Pr=\frac{\upsilon}{\alpha}. \end{aligned} \qquad (5)$$

Where *u, v* are velocities along *x* and *y* direction, *r* is the local radius of the cone, *x* and *y* are the spatial coordinates, t' is the time, *g* is the acceleration due to gravity, *β* is the volumetric thermal expansion, T' is the temperature, C' is the concentration, *υ* is the kinematic viscosity, *α* is the thermal diffusivity, Gr_L is the Grashoff number based on L.

Equations (1), (2) and (3) are reduced to the following non-dimensional form:

$$\frac{\partial(UR)}{\partial X}+\frac{\partial(VR)}{\partial Y}=0,\ \text{or}\ \left(\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}+\frac{U}{X}=0\right) \qquad (6)$$

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=T\cos\phi+\frac{\partial^2 U}{\partial Y^2}, \qquad (7)$$

$$\frac{\partial T}{\partial t}+U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}=\frac{1}{\Pr}\frac{\partial^2 T}{\partial Y^2}. \qquad (8)$$

The corresponding initial and boundary conditions in non-dimensional quantities are given by

$$\begin{aligned} t \le 0:&\ U=0,\ V=0,\ T=0, \quad \text{for all } X \text{ and } Y,\\ t>0:&\ U=0,\ V=0,\ T=X^n, \ \text{at}\ Y=0,\\ &\ U=0,\ T=0, \quad \text{at}\ X=0,\\ &\ U\to 0,\ T\to 0, \quad \text{as}\ Y\to\infty. \end{aligned} \qquad (9)$$

Where U and V are dimensionless velocity in X and Y direction, R is the dimensionless local radius, L is the reference length, Pr is the prandtl number, Sc is the Schmidt number, t is the dimensionless time.

Solution procedure

The unsteady nonlinear coupled partial differential equations (6-8) with the initial and boundary conditions (9) are solved using the new method, Network Simulation Method. The discretization of the boundary-layer equations is based on the difference-finite formulation, and only discretization of the spatial co-ordinates is necessary, time remaining as a real continuous variable.

Based up on these equations an electrical network circuit is designed. Electric analogy is applied in which the variable voltage (V) is equivalent to velocities (U, V) and temperature (T); the variable electric current (J) is equivalent to the velocity fluxes and temperature fluxes.

or each dimensionless boundary layer equation, two circuits are developed .whole network is converted into a suitable program that is solved by a computer code (electric circuits simulator), Pspice [14]. The Time interval required for the convergence is not a prerequisite since the code Pspice does this work with sophisticated mathematical algorithms, which are common for most currently used numerical methods.

Design of the Network Model:

Taking semi-infinite vertical cone slant height as $L = 1$, it is considered a rectangular region with X varying from 0 to 1 and Y varying from 0 to $Y_{max} = 20$, where $X = L$ corresponds to the slant height of the vertical cone and Y_{max} is regarded as ∞, where Y_{max} lies outside the momentum, thermal and species boundary layers. The region of integration is considered as a rectangle with mesh sizes $\Delta X = 0.25$ and $\Delta Y = 0.25$.

The design of the network model is as follows. The finite difference differential equations resulting from dimensionless continuity, momentum balance and energy balance equations[6,7,8] by applying the electrical analogy together with the Kirchhoff's law are

$$\left(U_{i+\Delta X,j}-U_{i-\Delta X,j}\right)/\Delta X+\left(V_{i,j}-V_{i,j-\Delta Y}\right)/\left(\Delta Y/2\right)+U_{ri,j}/\left(\mathrm{i}\Delta X/2\right)=0 \qquad (10)$$

$$\begin{aligned} &\Delta Y\, dU_{i,j}/dt+\Delta Y\, U_{i,j}\left(U_{i+\Delta X,j}-U_{i-\Delta X,j}\right)/\Delta X+V_{i,j}\left(U_{i,j+\Delta Y}-U_{i,j-\Delta Y}\right)\\ &=\left(U_{i,j-\Delta Y}-U_{i,j}\right)/\left(\Delta Y/2\right)-\left(U_{i,j}-U_{i,j+\Delta Y}\right)/\left(\Delta Y/2\right)+\Delta Y\, T_{i,j}\cos\phi \end{aligned} \qquad (11)$$

$$\begin{aligned} &\Delta Y\, Pr\, dT_{i,j}/dt+\Delta Y\, Pr\, U_{i,j}\left(T_{i+\Delta X,j}-T_{i-\Delta X,j}\right)/\Delta X+Pr\, V_{i,j}\left(T_{i,j+\Delta Y}-T_{i,j-\Delta Y}\right)\\ &=\left(T_{i,j-\Delta Y}-T_{i,j}\right)/\left(\Delta Y/2\right)-\left(T_{i,j}-T_{i,j+\Delta Y}\right)/\left(\Delta Y/2\right) \end{aligned} \qquad (12)$$

Each term of the difference equations [(11)-(12)] is considered as an electrical current, and is written as the combinations of Resistors, Capacitors and Generators as derived in [17]. More rigorous step-by-step numerical analysis is described by [16,17]. Fig 1A, 1B shows the network model corresponding to the equations [(11)-(12)].

The finite difference equation corresponding to equation (10) is

$$V_{i,j} = \left(U_{i-\Delta X,j} - U_{i+\Delta X,j}\right)\Delta Y / \left(2\,\Delta X\right) + U_{i,j}\Delta Y / \left(i\Delta X\right) - V_{i,j-\Delta Y} \tag{14}$$

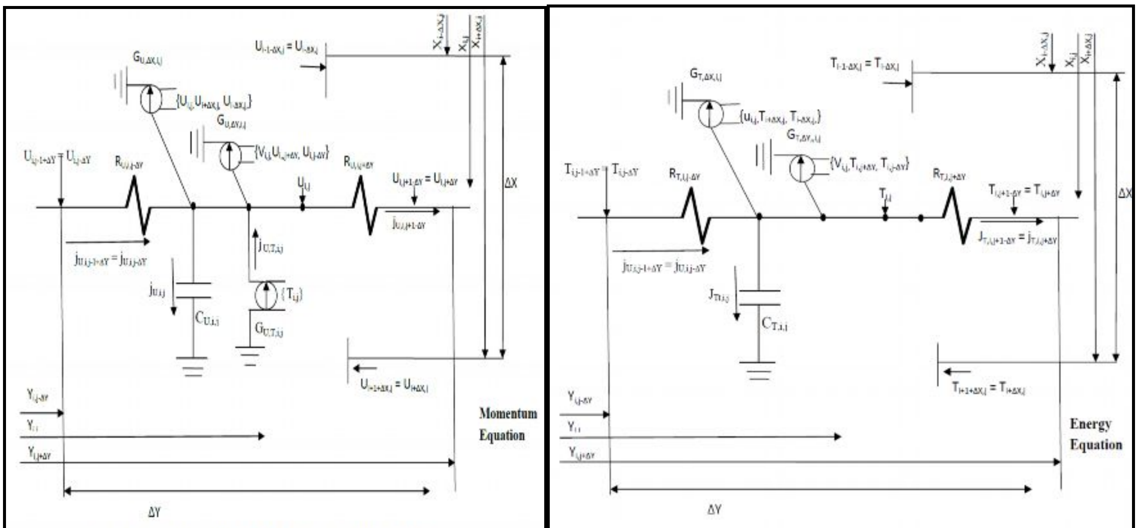

Fig. A. Network model of the control volume – Momentum equation

Fig. B. Network model of the control volume – Energy equation

Above Equations [(11) – (12)] can be written in the form of Kirchhoff's law as

$$\begin{aligned} &j_{U,\,i,\,j+\Delta Y} - j_{U,\,i,\,j-\Delta Y} - j_{UT,\,i,\,j} + j_{Ux,\,i,\,j} + j_{Uy,\,i,\,j} + j_{Ut,\,i,\,j} = 0 \\ &j_{T,\,i,\,j+\Delta Y} - j_{T,\,i,\,j-\Delta Y} + j_{Tx,\,i,\,j} + j_{Ty,\,i,\,j} + j_{Tt,\,i,\,j} = 0 \end{aligned} \tag{15}$$

Results and Discussion

In order to prove the accuracy of our numerical results, the present results in steady state at $X = 1.0$, $\Pr = 0.7$, $\eta = Y$ and considering $Gr_L^* = Gr_L \cos\phi = \dfrac{g\beta\cos\phi\left(T_w' - T_\infty'\right)L^3}{\upsilon^2}$ are compared with available similarity solutions in the literature. In Fig 2 the velocity and temperature profiles of isothermal and non-isothermal vertical cone are compared with similarity solutions of [4] in steady state and found to be in excellent agreement. Finally, results of [9] are for incompressible fluid same as those of [4] results. Hence, present results well agree with the results of [9] for incompressible fluid.

In Figs. 3-8, transient velocity and temperature profiles are shown at $X = 1.0$, with various parameters $\Pr$, n and ϕ. The values of t with star (*) symbols denote the time taken to reach steady state. In Fig. 3, transient velocity profiles are shown for different angles with $\Pr = 0.71$ and $n = 0.2$. When ϕ increases, near the cone apex, it leads to decrease in the impulsive force along the cone surface. Hence, the difference between temporal maximum velocity values and steady state values decreases with increasing the values of ϕ. The tangential component of buoyancy force reduces as the semi vertical angle increases. This causes the velocity to reduce as angle ϕ increases. The momentum boundary layer becomes thick, and the time taken to reach steady state increases for increasing ϕ. In Fig. 4, transient temperature profiles are shown for different angles with $\Pr = 0.71$ and $n = 0.2$. It is observed the temperature and boundary layer thickness increase with increasing ϕ. The difference between temporal maximum temperature values and steady state values decrease with increasing ϕ.

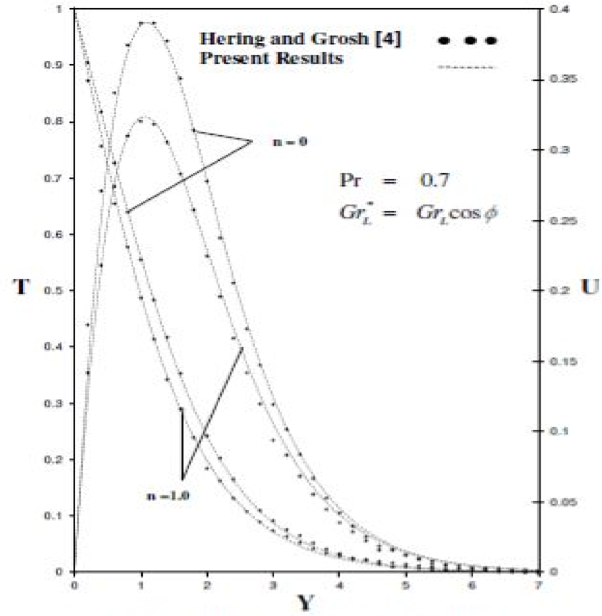

Fig. 2 Comparison of steady state temperature and velocity profiles at X = 1.0

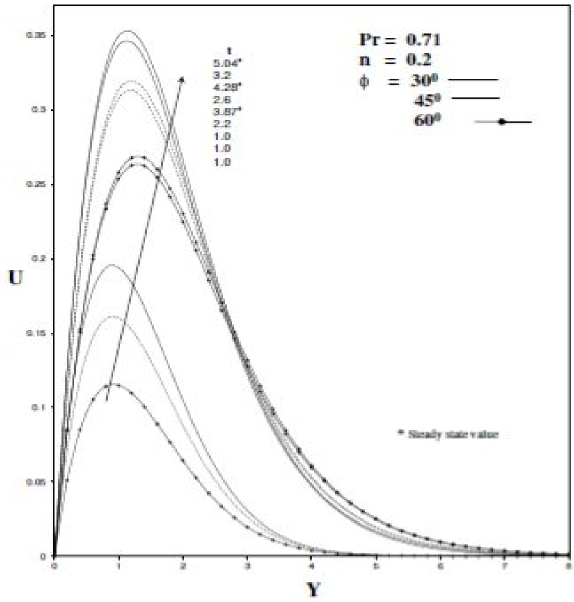

Fig. 3. Transient velocity profiles at X = 1.0 for different values of φ

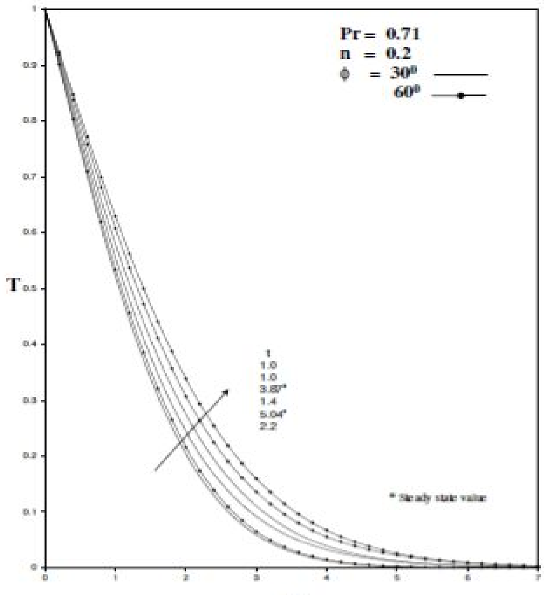

Fig. 4. Transient temperature profiles at X = 1.0 for different values of φ

In Figs. 5 and 6, transient velocity and temperature profiles are plotted for various values of $\Pr$ with $\phi = 15^0$ and $n = 0.2$. Viscous force increases and thermal diffusivity reduces with increasing $\Pr$, causes a reduction in the velocity and temperature as expected. It is observed from the figures that the difference between temporal maximum

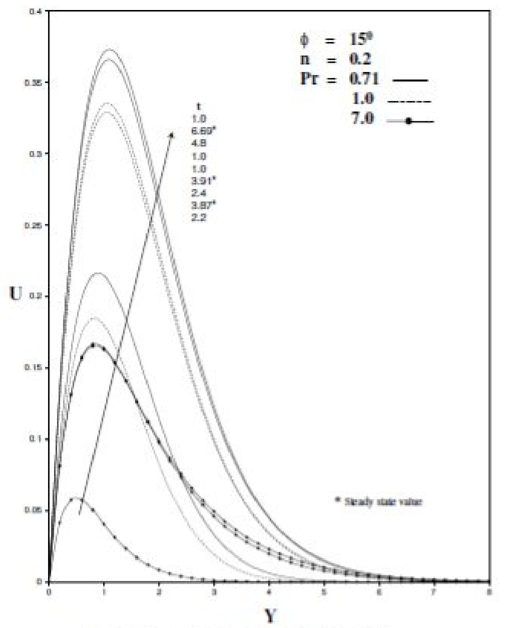

Fig. 5. Transient velocity profiles at X = 1.0 for different values of Pr

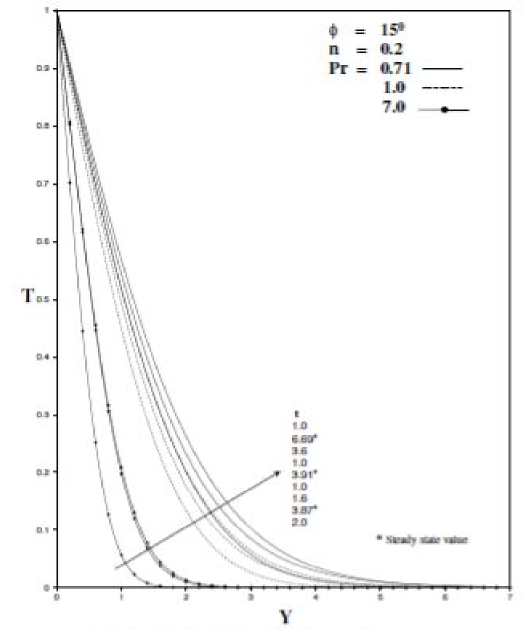

Fig. 6. Transient temperature profiles at X = 1.0 for different values of Pr

values and steady state values are reduced when Pr increases. It is also noticed the time taken to reach steady state increases and thermal boundary layer thickness reduces with increasing Pr . It is also clear from the Fig. 5, the momentum boundary layer thickness increases with the increase of Pr from unity.

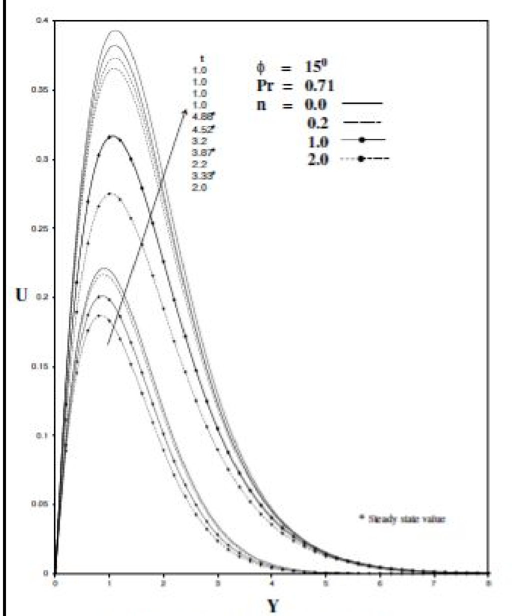

Fig. 7. Transient velocity profiles at X = 1.0 for different values of n

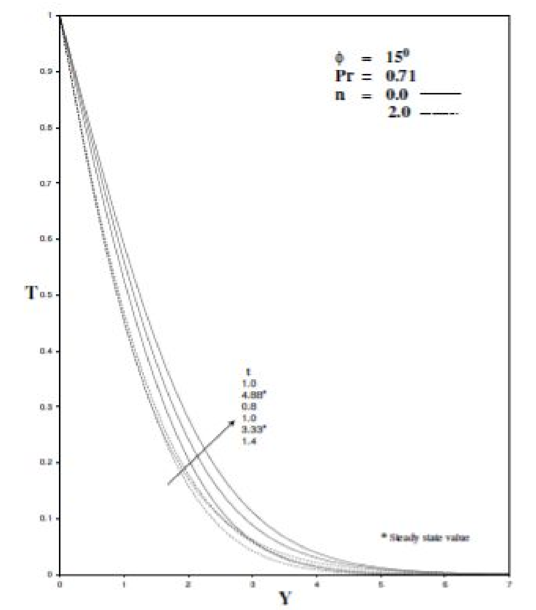

Fig. 8. Transient temperature profiles at X = 1.0 for different values of n

In Figs. 7 and 8, transient velocity and temperature profiles are shown for various values of n with $\Pr = 0.71$ and $\phi = 15^0$. Impulsive forces are reduced along the surface of the cone near the vertex for increasing values of n . Due to this, the difference between temporal maximum values and steady state values reduce. It is also observed that as n increases, velocity and temperature reduce and the time taken to reach steady state value increases.

Conclusion

This paper deals with the flow over an unsteady non-isothermal vertical cone. The dimensionless governing boundary layer equations are solved by Network Simulation Method. Present results are compared with available results in literature and are found to be in good agreement. The following conclusions are made:

1. The velocity reduces when the parameters ϕ, Pr, n are increased and Temperature increases with increasing ϕ and decreasing Pr, n values.
2. Momentum boundary layers become thick when ϕ and Pr are increased and Thermal boundary layer becomes thin when ϕ is reduced and Pr is increased.
3. The time taken to reach steady state increases with increasing ϕ, Pr and n .
4. The difference between temporal maximum velocity value and steady state value becomes more when ϕ, Pr and n are decreased.
5. Decreasing ϕ or increasing Pr, n reduces the difference between temporal maximum temperature values and steady state values.

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